CALCULATION OF FRICTION HEAD LOSS IN PLUG FLOW OF GAS – LIQUID MIXTURE IN TUBES

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The resistance coefficient is derived from the hydrodynamic equations for multiphase liquids by using an empirical relationship for the dynamic velocity and general semi-empirical considerations.

The resistance of gas–liquid plug flow has been fairly well investigated experimentally in [1-4]. None of them give a soundly based method of calculating the resistance in the case of plug flow of a gas –liquid mixture. In the present paper the equations of motion of multicomponent mixtures, derived in [5], are used to find the hydraulic resistance coefficient for plug flow of a gas–liquid mixture. By plug flow we mean flows which have no distinct interface (plug flows, plug flows with foam formation, etc.).

The equation of motion of a multicomponent mixture after simplification for the case of a unidimensional (in relation to the mean values) gas–liquid flow has the form [5]

$$-\frac{dp}{dx}+[\rho_2+\bar{\varphi}_2(\rho_2-\rho_1)]\,g\cos(x_1g)+\frac{d\bar{\sigma}}{dy}-\frac{d}{dy}(\rho_l\bar{\varphi}_l\overline{u'_{lx}u'_{ly}}$$
$$+\rho_l\bar{u}_{lx}\overline{\varphi'_l u'_{ly}}+\rho_l\bar{u}_{ly}\overline{\varphi'_l u'_{lx}}+\rho_l\overline{\varphi'_l u'_{lx}u'_{ly}})=0. \quad (1)$$

The term in the parentheses in the equation of motion is the turbulent stress of the gas–liquid flow. In addition to the moments associated with one-phase flow, the equation of motion (1) has mixed moments between the fluctuations of the velocities and concentrations of the two phases.

We put moments of type $\rho_l\bar{\varphi}_l\overline{u'_{lx}u'_{ly}}$, as in one-phase flow [6], in the following form:

$$\bar{\varphi}_1\rho_1\overline{u'_{1x}u'_{1y}}=\bar{\varphi}_1\rho_1 N_{yy}\frac{d\bar{u}_{1x}}{dy}, \quad (2)$$

$$\bar{\varphi}_2\rho_2\overline{u'_{2x}u_{2y}}=\bar{\varphi}_2\rho_2 N_{yy}\frac{d\bar{u}_{2x}}{dy}. \quad (3)$$

We assume here that the true gas content is constant over the entire cross section of the tube.

We express the mixed moments between the velocity and concentration fluctuations in the two phases in terms of moments of the type $\rho_l\bar{\varphi}_l\overline{u'_{lx}u'_{ly}}$ by means of the relations

$$\rho_1(\bar{u}_{1x}\overline{\varphi'_1u'_{1y}}+\overline{\varphi'_1u'_{1x}}\bar{u}_{1y}+\overline{\varphi'_1u'_{1x}u'_{1y}})=f_1\bar{\varphi}_1\rho_1N_{yy}\frac{d\bar{u}_{1x}}{dy}, \quad (4)$$

$$\rho_2(\bar{u}_{2x}\overline{\varphi'_2u'_{2y}}+\overline{\varphi'_2u'_{2x}}\bar{u}_{2y}+\overline{\varphi'_2u'_{2x}u'_{2y}})=f_2\bar{\varphi}_2\rho_2N_{yy}\frac{d\bar{u}_{2x}}{dy}, \quad (5)$$

where f_1 and f_2 are functions which can be determined experimentally. Figure 1 shows the dynamic velocity w_* of the mixture as a function of the discharge gas content β for different Froude numbers

$$w_*=\sqrt{\frac{\tau_0\bar{v}_1^2}{\rho_1\bar{\varphi}_1\bar{v}_1^2+\rho_2\bar{\varphi}_2\bar{v}_2^2}}\approx\sqrt{\frac{\tau_0}{\rho_c}}. \quad (6)$$

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It has been experimentally established that the last two functions in expression (6) are numerically equal. The tangential stress on the tube wall is given by the equation [7]

$$\tau_0 = \frac{d\bar{p}}{dx} \cdot \frac{R}{2} . \tag{7}$$

The dynamic velocity of the mixture is connected with the discharge gas content and Froude number by the following empirical expression:

$$w_* = w_{*0} + w_{*0} (\beta_2 - \bar{\varphi}_2) \exp\left(-\frac{k_1}{\mathrm{Fr}}\right) \quad (k_1 = 0.4), \tag{8}$$

where $w_{*0} = u_{*1}$ is the dynamic velocity at $\beta_2 = 0$.

For subsequent calculations we need to express the true velocity of one phase in terms of the true velocity of the other.

The velocity of the liquid phase is expressed in terms of the velocity of the mixture by the following relationship:

$$\bar{u}_1 = \bar{u}_{\mathrm{m}} \frac{\beta_1}{\bar{\varphi}_1} , \tag{9}$$

where

$$\bar{u}_{\mathrm{m}} = \bar{u}_1\bar{\varphi}_1 + \bar{u}_2\bar{\varphi}_2, \quad \beta_1 = 1 - \beta_2. \tag{10}$$

Substituting equation (10) in expression (9), we obtain:

$$\bar{u}_1 = (\bar{u}_1\bar{\varphi}_1 + \bar{u}_2\bar{\varphi}_2) \frac{\beta_1}{\bar{\varphi}_1} = \bar{u}_1\beta_1 + \frac{\bar{u}_2\beta_2\bar{\varphi}_2}{\bar{\varphi}_1} , \tag{11}$$

from which, using the relationship

$$1 - \beta_1 = \beta_2, \tag{12}$$

we have

$$\bar{u}_1\beta_2 = \bar{u}_2 \frac{\bar{\varphi}_2\beta_1}{\bar{\varphi}_1} \tag{13}$$

and we finally obtain the following formulas:

$$\bar{u}_1 = \bar{u}_2 \frac{\bar{\varphi}_2\beta_1}{\bar{\varphi}_1\beta_2} , \tag{14}$$

$$\bar{u}_2 = \bar{u}_1 \frac{\bar{\varphi}_1\beta_2}{\bar{\varphi}_2\beta_1} . \tag{15}$$

We assume that the moments of type $\bar{\varphi}_l \rho_l \overline{u'_{lx} u'_{ly}}$ are proportional to the dynamic velocity w_{*0}, and the mixed moments between the velocity and concentration fluctuations are proportional to $w_{*0}(\beta_2 - \bar{\varphi}_2) \exp(-k_1/\mathrm{Fr})$.

We express the moments of type $\bar{\varphi}_l \rho_l \overline{u'_{lx} u'_{ly}}$ and the mixed moments between the velocity and concentration fluctuations of the two phases in terms of the turbulent viscosity of the mixture and the gradients of the mean velocities of the corresponding phases:

$$\bar{\varphi}_1\rho_1\overline{u'_{1x} u'_{1y}} = \bar{\varphi}_1\rho_1 \varkappa w_{*0} y \frac{d\bar{u}_{1x}}{dy} , \tag{16}$$

$$\bar{\varphi}_2\rho_2\overline{u'_{2x} u'_{2y}} = \bar{\varphi}_2\rho_2 \varkappa w_{*0} y \frac{d\bar{u}_{2x}}{dy} , \tag{17}$$

$$\rho_1 (\overline{\varphi'_1 u'_{1x}}\, \bar{u}_{1y} + \overline{\varphi'_1 u'_{1y}}\, \bar{u}_{1x} + \overline{\varphi'_1 u'_{1x} u'_{1y}}) = \rho_1 \varkappa w_* y \frac{d\bar{u}_{1x}}{dy} (\beta_2 - \bar{\varphi}_1) e^{-k_1/\mathrm{Fr}}, \tag{18}$$

$$\rho_2 (\overline{\varphi'_2 u'_{2x}}\, \bar{u}_{2y} + \overline{\varphi'_2 u'_{2y}}\, \bar{u}_{2x} + \overline{\varphi'_2 u'_{2x} u'_{2y}}) = \rho_2 \varkappa w_{*0} y \frac{d\bar{u}_{2x}}{dy} (\beta_2 - \bar{\varphi}_2) e^{-k_1/\mathrm{Fr}}. \tag{19}$$

The total stress in the flow, equal to the sum of the viscous and turbulent stresses, is

$$\tau = \bar{\varphi}_1\mu_1 \frac{d\bar{u}_{1x}}{dy} + \bar{\varphi}_2\mu_2 \frac{d\bar{u}_2}{dy} + \bar{\varphi}_1 \varkappa w_{*0} y \rho_1 \frac{d\bar{u}_{1x}}{dy}\left[1 + (\beta_2 - \bar{\varphi}_2) e^{-k_1/\mathrm{Fr}}\right] + \bar{\varphi}_2 \rho \varkappa w_{*0} y \frac{d\bar{u}_{2x}}{dy}\left[1 + (\beta_2 - \bar{\varphi}_2) e^{-\kappa_1/\mathrm{Fr}}\right]. \quad (20)$$

Replacing the velocity of the gas phase in this expression by the velocity of the liquid phase by using formulas (14) and (15), dividing it by the density of the mixture

$$\rho_m = \rho_1\bar{\varphi}_1 + \rho_2\bar{\varphi}_2 \quad (21)$$

and putting

$$\frac{\bar{\varphi}_1\mu_1}{\rho_m} + \frac{\bar{\varphi}_2\mu_2}{\rho_m} \cdot \frac{\bar{\varphi}_1\beta_2}{\bar{\varphi}_2\beta_1} = \nu_g,$$
$$\frac{\rho_1\bar{\varphi}_1}{\rho_m}\left[1 + (\beta_2 - \bar{\varphi}_2) e^{-k_1/\mathrm{Fr}}\right] + \frac{\rho_2\bar{\varphi}_2}{\rho_m} \cdot \frac{\bar{\varphi}_1\beta_2}{\bar{\varphi}_2\beta_1}\left[1 + (\beta_2 - \bar{\varphi}_2) e^{-k_1/\mathrm{Fr}}\right] = B, \quad (22)$$

we obtain the following differential equation for the gradient of the mean velocity of the liquid phase:

$$w_{*0}^2 = (\nu_g + \varkappa w_{*0} y B) \frac{d\bar{u}_{1x}}{dy}. \quad (23)$$

Integrating it, we have

$$\bar{u}_{1x} = \frac{w_{*0}}{\varkappa B}\left[1 + (\beta_2 - \bar{\varphi}_2) e^{-k_1/\mathrm{Fr}}\right] \ln\left(1 + \frac{\varkappa w_{*0} y B}{\nu_g}\right) \quad (24)$$

at $y = k$

$$\bar{u}_{1x} = \bar{u}_{10}, \quad (25)$$

where $\bar{u}_{10}$ is the velocity of the liquid phase at the level of the mean roughness height. It was assumed in the integration that $\bar{\varphi}_1$ and $\bar{\varphi}_2$ do not vary over the cross section of the tube. Determining the constant of integration from the boundary condition (25) and substituting it in equation (24), we obtain

$$\frac{\bar{u}_{1x}}{w_{*0}} = \frac{\left[1 + (\beta_2 - \bar{\varphi}_2) e^{-k_1/\mathrm{Fr}}\right]}{\varkappa B} \ln \frac{1 + \frac{\varkappa w_{*0} y B}{\nu_g}}{1 + \frac{\varkappa w_{*0} B k}{\nu_g}} + b, \quad (26)$$

where $b = \bar{u}_{10}/w_{*0}$ is a function which can be determined experimentally.

To find a theoretical equation for the pressure gradient we replace the dynamic velocity in Eq. (26) by its value given by expression (6). We then have

$$\sqrt{\frac{\rho_1\bar{\varphi}_1\bar{v}_1^2 + \rho_2\bar{\varphi}_2\bar{v}_2^2}{\tau_0}} = \frac{1 + (\beta_2 - \bar{\varphi}_2) e^{-k_1/\mathrm{Fr}}}{\varkappa B} \ln \frac{1 + \frac{\varkappa w_{*0} B y_v}{\nu_g}}{1 + \frac{\varkappa w_{*0} B k}{\nu_g}} + b = \frac{1}{2\sqrt{\lambda_m}}, \quad (27)$$

where y_v is the coordinate of the mean velocity of the liquid phase. Replacing the tangential stress in this expression by the pressure gradient from (7) and squaring the obtained relationship we obtain a theoretical equation for the pressure gradient in the plug regime

$$\frac{d\bar{p}}{dx} = -\lambda_m \frac{(\rho_1\bar{\varphi}_1\bar{v}_1^2 + \rho_2\bar{\varphi}_2\bar{v}_2^2)}{2d}. \quad (28)$$

We find the hydraulic resistance coefficient from formula (27)

$$\frac{1}{\sqrt{\lambda_m}} = \frac{2\left[1 + (\beta_2 - \bar{\varphi}_2) e^{-k_1/\mathrm{Fr}}\right]}{\varkappa B} \ln \frac{1 + \frac{\varkappa w_{*0} B y_v}{\nu_g}}{1 + \frac{\varkappa B w_{*0} k}{\nu_g}} + b. \quad (29)$$

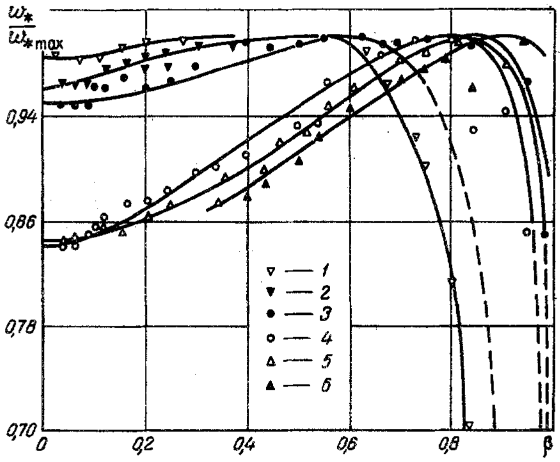

Fig. 1

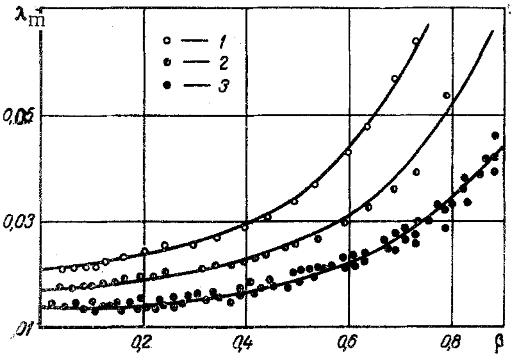

Fig. 2

Fig. 1. Dynamic velocity of gas–liquid flow as function of discharge gas content and Froude number of mixture (tube with d = 100 mm): 1) Fr = 0.4; 2) 0.8; 3) 2.0; 4) 4.0; 5) 8; 6) 20.

Fig. 2. Comparison of theoretical and experimental data for hydraulic resistance coefficient: 1) Fr = 0.4; 2) 2; 3) 4-20 (d = 50 and 10 mm).

We neglect the 1 in the numerator of the logarithm and from Eq. (29) we obtain

$$\frac{1}{\sqrt{\lambda_m}} = \frac{2\left[1+(\beta_2-\bar{\varphi}_2)\,e^{-k_1/\mathrm{Fr}}\right]}{\varkappa B}\ln\left[\frac{\nu_g}{\varkappa w_{*0} B y_v}+\frac{k}{\varkappa R}+b\right]. \quad (30)$$

We replace the dynamic velocity in the denominator of the logarithmic expression by the mean velocity of the liquid phase, and the hydraulic resistance coefficient by using the formula

$$w_{*0} = u_{*1x} = v_1\sqrt{\frac{\lambda_1}{8}} \quad (31)$$

and after manipulation of the expression under the logarithm sign we have

$$\frac{1}{\sqrt{\lambda_m}} = -\frac{2\left[1+(\beta_2-\bar{\varphi}_2)\,e^{-k_1/\mathrm{Fr}}\right]}{\varkappa B}\ln\left\{\frac{\sqrt{8}}{\varkappa\sqrt{\lambda_1}}\left[\frac{\bar{\varphi}_1}{\mathrm{Re}_1(\bar{\varphi}_1+\dot{\rho}\bar{\varphi}_2)}+\frac{\dot{\rho}\bar{\varphi}_2}{\mathrm{Re}_2(\bar{\varphi}_1+\bar{\varphi}_2\dot{\rho})}\left(\frac{\bar{\varphi}_1\beta_2}{\bar{\varphi}_2\beta_1}\right)^2\right]+\frac{k}{\varkappa R}+b\right\}. \quad (32)$$

We put formula (32) in the following form:

$$\frac{1}{\sqrt{\lambda_m}} = -A_1\frac{\left[1+(\beta_2-\bar{\varphi}_2)\,e^{-k_1/\mathrm{Fr}}\right]}{B}\lg\left\{\left[\frac{A_2}{\sqrt{\lambda_1}}\left(\frac{\bar{\varphi}_1}{\mathrm{Re}_1(\bar{\varphi}_1+\dot{\rho}\bar{\varphi}_2)}+\frac{\dot{\rho}\bar{\varphi}_2}{\mathrm{Re}_2(\bar{\varphi}_1+\dot{\rho}\bar{\varphi}_2)}\left(\frac{\bar{\varphi}_1\beta_2}{\bar{\varphi}_2\beta_1}\right)^2\right)+\frac{A_3 k}{d}\right]e^b\right\}. \quad (33)$$

Formula (33) with $\beta = 0$ becomes the Colebrook formula for a one-phase flow with coefficients A_1, A_2, and A_3, equal to 2, 2.5, 1/2.8, respectively [8]. Substituting these coefficients in relationship (33), we obtain

$$\frac{1}{\sqrt{\lambda_m}} = -2\,\frac{\left[1+(\beta-\bar{\varphi}_2)\,e^{-k_1/\mathrm{Fr}}\right]}{B}\lg\left\{\left[\frac{2.5}{\sqrt{\lambda_1}}\left(\frac{\bar{\varphi}_1}{\mathrm{Re}_1(\bar{\varphi}_1+\dot{\rho}\bar{\varphi}_2)}+\frac{\dot{\rho}\bar{\varphi}_2}{\mathrm{Re}_2(\bar{\varphi}_1+\dot{\rho}\bar{\varphi}_2)}\left(\frac{\bar{\varphi}_1\beta_2}{\bar{\varphi}_2\beta_1}\right)\right)+\frac{k}{2{,}8d}\right]e^b\right\}. \quad (34)$$

Function b was determined by experiment and was

$$b = \frac{\sqrt{\beta_2-\bar{\varphi}_2}}{1-\beta_2}+\frac{0.2}{\mathrm{Fr}}\sqrt{\beta_2}. \quad (35)$$

Substituting the value of function b in the expression for the hydraulic resistance coefficient we have

$$\frac{1}{\sqrt{\lambda_m}} = -2\,\frac{\left[1+(\beta-\bar{\varphi}_2)\,e^{-k_1/\mathrm{Fr}}\right]}{B}\lg\left\{\left[\frac{2{,}5}{\sqrt{\lambda_1}}\left(\frac{\bar{\varphi}_1}{\mathrm{Re}_1(\bar{\varphi}_1+\dot{\rho}\bar{\varphi}_2)}\right.\right.\right.$$

$$\left.\left.\left.+\frac{\dot{\rho}\bar{\varphi}_2}{\mathrm{Re}_2(\bar{\varphi}_1+\dot{\rho}\bar{\varphi}_2)}\left(\frac{\bar{\varphi}_1\beta_2}{\bar{\varphi}_2\beta_1}\right)^2\right)+\frac{k}{2{,}8d}\right]\exp\left(\frac{\sqrt{\beta_2-\bar{\varphi}_2}}{1-\beta_2}+\frac{0{,}2}{\mathrm{Fr}}\sqrt{\beta_2}\right)\right\}, \quad (36)$$

where

$$B = \frac{\rho_1\overline{\varphi}_1}{\rho_m}\left[1+(\beta_2-\overline{\varphi}_2)e^{-k_1/\mathrm{Fr}}\right]+\frac{\rho_2\overline{\varphi}_2}{\rho_m}\left(\frac{\overline{\varphi}_1\beta_2}{\overline{\varphi}_2\beta_1}\right)^2\left[1+(\beta_2-\overline{\varphi}_2)e^{-k_1/\mathrm{Fr}}\right].$$

A comparison of formula (36) with the experimental data of various authors is shown in Fig. 2. It is evident from Fig. 2 that the theoretical and experimental data agree satisfactorily throughout the range of variation of the Froude number of the mixture and the discharge gas content.

Thus, by applying the general equation of motion of a multicomponent mixture and using it to close the semiempirical theory of turbulence we have derived theoretical equations for the friction head loss in plug flow of a gas–liquid mixture.

NOTATION

$d\overline{p}/dx$	is the pressure gradient;
$\rho_{1(2)}$	is the density of liquid (gas) phase;
$\overline{\varphi}_{1(2)} = F_{1(2)}/F$	is the true gas content;
$F_{1(2)}$	is the cross-sectional area of the tube occupied by liquid (gas) phase;
F	is the cross-sectional area of the whole tube;
$\overline{\sigma}_{1(2)} = \mu_{1(2)}(d\overline{u}_{1(2)}/dy)$	is the viscous stress of liquid (gas) phase;
$\nu_{1(2)} = \mu_{1(2)}/\rho_{1(2)}$	is the kinematic viscosity of liquid (gas) phase;
$\overline{u}_{1(2)}$	is the mean (at point) velocity of liquid (gas) phase;
$u'_{1(2)x(y)}$	are the fluctuational velocity of liquid (gas) phase along x and y axes;
$\varphi'_{1(2)}$	is the fluctuation of liquid (gas) concentration;
N_{yy}	is the turbulent viscosity;
$w_* = \sqrt{\tau_0/\rho_m}$	is the dynamic velocity of the mixture;
τ_0	is the tangential stress of the mixture of the tube wall;
$\rho_m = \rho_1\overline{\varphi}_1 + \rho_2\overline{\varphi}_2$	is the density of mixture;
R	is the tube radius;
$\beta_2 = Q_2/(Q_1 + Q_2)$	is the discharge gas content;
$Q_{1(2)}$	is the flow rate of liquid (gas) phase;
$\varkappa$	is the Karman constant;
$\mathrm{Fr} = \overline{w}_m^2/gd$	is the Froude number of the mixture;
$w_m = (Q_1 + Q_2)/F$	is the velocity of mixture;
k	is the mean roughness height of tube wall;
λ_1	is the resistance coefficient of liquid phase at $\beta = 0$;
λ_m	is the resistance coefficient of mixture;
l	is the subscript for summation for each phase ($l = 1, 2$).

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